2/3 4 1/2

a 1

$$\begin{cases} u_t = \Delta_2 u + \sin t \cdot \sin x \cdot \sin y & -\infty < x, y < +\infty, \quad t > 0 \\ u|_{t=0} = 1 \end{cases}$$

$$u = v + w$$

$$\begin{cases} v_t = \Delta_2 v \\ v|_{t=0} = 1, \end{cases} \qquad \begin{cases} w_t = \Delta_2 w + \sin t \cdot \sin x \cdot \sin y \\ w|_{t=0} = 0 \end{cases}$$

$\exists\, v = v(t)$

$\exists\, w = T(t) \cdot \sin x \cdot \sin y$

$$\begin{cases} T' \cdot \sin x \cdot \sin y = -2T \sin x \cdot \sin y + \sin t \cdot \sin x \cdot \sin y \\ T(0) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} v_t = 0 \\ v|_{t=0} = 1 \end{cases}$$

$$\begin{cases} T' = -2T + \sin t \\ T(0) = 0 \end{cases}$$

$\Rightarrow v = 1$

1) $T' = 2T \Rightarrow T = C \cdot e^{-2t}$

2) $T_ч = \alpha \cos t + \beta \sin t$

$-\alpha \sin t + \beta \cos t = -2\alpha \cos t - 2\beta \sin t + \sin t$

$$\begin{cases} -\alpha = -2\beta + 1 \\ \beta = -2\alpha \end{cases} \Rightarrow \begin{cases} -\alpha + 2\beta = 1 \\ -2\alpha + \beta = 0 \end{cases} \Rightarrow \begin{cases} \alpha = -1/5 \\ \beta = 2/5 \end{cases}$$

$\Rightarrow T = Ce^{-2t} - 1/5 \cos t + 2/5 \sin t$

$T(0) = C - 1/5 = 0 \Rightarrow C = 1/5$

$\Rightarrow w = \sin x \cdot \sin y (1/5 e^{-2t} - 1/5 \cos t + 2/5 \sin t)$

Ответ: $\boxed{u = 1 + \sin x \cdot \sin y (1/5 e^{-2t} - 1/5 \cos t + 2/5 \sin t)}$

№2

$$\begin{cases} u_t = 1/2\,\Delta_2 u + t\sin x\cdot\cos y & -\infty < x, y < \infty \\ u|_{t=0} = xy & t > 0 \end{cases}$$

$$u(x,y,t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{1}{2a\sqrt{\pi t}}\right)^2 e^{-\frac{(x-\xi)^2+(y-\eta)^2}{4a^2t}}\cdot\xi\eta\cdot d\xi\, d\eta +$$

$$+\int_0^t\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{1}{2a\sqrt{\pi(t-\tau)}}\right)^2\cdot e^{-\frac{(x-\xi)^2+(y-\eta)^2}{4a^2(t-\tau)}}\cdot\tau\cdot\sin\xi\cdot\cos\eta\, d\xi\, d\eta\cdot d\tau =$$

$$= I_1(x,t)\cdot I_2(x,t) + \int_0^t \tau I_3\cdot I_4\cdot d\tau$$

$$I_1 = \int_{-\infty}^{+\infty}\frac{1}{2a\sqrt{\pi t}}e^{-\frac{(x-\xi)^2}{4a^2t}}\xi\, d\xi = \left\{\begin{aligned} a &= \sqrt{\tfrac{1}{2}} \\ z &= \frac{\xi-x}{2a\sqrt{t}}\end{aligned}\right\} = \int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi t}}\cdot e^{-z^2}(z\sqrt{2t}+x)\sqrt{2t}\, dz$$

$$= \int_{-\infty}^{+\infty} e^{-z^2} z\cdot\frac{\sqrt{2t}}{\sqrt{\pi}}\, dz + \int_{-\infty}^{+\infty}\frac{e^{-z^2}x}{\sqrt{\pi}}\, dz = 0 + x = x$$

$I_2 = y$ аналогично

$$I_3 = \int_{-\infty}^{+\infty}\frac{1}{2a\sqrt{\pi(t-\tau)}}e^{-\frac{(x-\xi)^2}{4a^2(t-\tau)}}\sin\xi\, d\xi = \left\{\begin{aligned} a &= \sqrt{\tfrac{1}{2}} \\ z &= \frac{\xi-x}{2a\sqrt{t-\tau}}\end{aligned}\right\} = \int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi(t-\tau)}}e^{-z^2}\sin(z\sqrt{2(t-\tau)}+x)\sqrt{2(t-\tau)}\, dz$$

$$= \int_{-\infty}^{+\infty}\frac{1}{\sqrt{\pi}}e^{-z^2}\cdot(\sin(z\sqrt{2(t-\tau)})\cdot\cos x + \sin x\cdot\cos(z\sqrt{2(t-\tau)}))\, dz =$$

$$= \left\{\int_{-\infty}^{+\infty} e^{-z^2}\cos 2cz\, dz = \sqrt{\pi}e^{-c^2},\ \int_{-\infty}^{+\infty} e^{-z^2}\sin 2cz\, dz = 0\right\} =$$

$$= \sin x\cdot e^{-\left(\frac{\sqrt{2(t-\tau)}}{2}\right)^2} = \sin x\, e^{\frac{\tau-t}{2}}$$

$I_4 = \cos y\, e^{\frac{\tau-t}{2}}$ аналогично, только cos раскладывается в сумму, так что остаётся $\cos y$, а не $\sin y$

$$\int_0^t \tau\cdot\sin x\, e^{\frac{\tau-t}{2}}\cdot\cos y\, e^{\frac{\tau-t}{2}}\, d\tau = \sin x\cdot\cos y\int_0^t \tau e^{\tau-t}\, d\tau =$$

$$= \sin x\cdot\cos y\left(\tau e^{\tau-t}\Big|_0^t - \int_0^t e^{\tau-t}\, d\tau\right) = \sin x\cdot\cos y\left(t - e^{\tau-t}\Big|_0^t\right) = (t + e^{-t} - 1)\sin x\cdot\cos y$$

Ответ: $\boxed{u = xy + \sin x\cdot\sin y\,(t - 1 + e^{-t})}$